**Concepts to Study**

1. Binary Operator
2. Group Theory – 4 axioms
3. Problems on Group Theory
4. Addition & Multiplication Modulo
5. Problems w.r.to modulo operations
6. Properties of Group
7. Semi – group
8. Sub – group
9. Necessary & sufficient condition for a sub – group.
10. P.T the set H = 2n is a sub – group of the additive group of integers.
11. If G is a group, then set N(a) is a subgroup of G.
12. S.T intersection of 2 subgroups is a sub – group & union of 2 sub – groups, need not be a sub – group.
13. 4 Theorems –
    1. If H & K are the subgroups of G; (HK)^-1 = K^-1\*H^-1
    2. If H is a subgroup of G; then H^-1 = H.
    3. A subset H of a group G is a subgroup iff H\*H^-1=H
    4. If H,K are any 2 subgroups of G; then HK is a subgroup iff;H.K=K.H.
14. If G is a group & ‘a’ belonging to G w.r.to multiplication is a fixed element of G; set H = (a^n ; n = G) is a subgroup of G.
15. If G is a group where (ab)^n = a^n\*b^n; for 3 consecutive integers n; for all a,b = G; S.T G must be abelian.
16. If ‘a’ & ‘x’ are any 2 elements of group G; P.T (xax^-1)^n = xa^nx^-1
17. Order of an element in a group.
18. Properties of order of element of a group.
19. If a & x are any 2 elements of group G; O(a) = O(xax^-1)
20. Prove that O(ab)=O(ba); where G is a group.
21. If a is any element of group G of order n; then a^m = e; for any integer m; iff n divides m. Given order of a is n. a^n = e.
22. Cyclic group
23. Properties of cyclic group.
24. P.T every abelian group need not be a cyclic group.
25. If ‘a’ is a generator of cyclic group G; O(a) = O(G)
26. Let G be a cyclic group of order ‘d’ & ‘a’ be a generator. The element a^k (k<d) is also a generator of ‘d’; iff O(k,d) = 1.
27. Every subgroup of a cyclic group is cyclic.
28. Left & Right coset of a group
29. Index of a subgroup
30. Lagrange’s Theorem
31. A finite group of prime order is cyclic & hence abelian.
32. Any group of order less than 6 is abelian.
33. Normal subgroup
34. A subgroup H of a group G is normal iff.
35. The intersection of any 2 normal subgroups of a group G is also normal.
36. A Cyclic group of order d has phi(d) generators.
37. Find no of generators of cyclic group of order.
38. Coset decomposition of a group.
39. Equivalence class.
40. If H is a subgroup of G; then [a] = Ha.
41. If H is a subgroup of G; then there exists a one to one correspondence between any 2 left(right) cosets of H in G.
42. There is one to one correspondence between set of all right cosets & set of all left cosets of a subgroup of a group.
43. Consequences of Lagrange’s Theorem.
    1. If G is a finite group, then O(a) divides O(G).
    2. A Finite group of prime order is cyclic & hence abelian.
    3. If G is a cyclic group of prime order; then G has no proper sub – groups.
44. Find all subgroups of cyclic group of 4th root of unity.
45. Find all subgroups of (Z18, +18)
46. A Subgroup H of a group G is normal iff every right coset of H in G is a left coset of H in G.
47. If H is a subgroup & K be a normal subgroup of a group G; then H intersection K is normal in H.
48. If H, K are 2 normal subgroups; then S.T the product of 2 normal subgroups is a subgroup.
49. If N is a normal subgroup of G & H is a subgroup of G; then NH = nh|
50. A subgroup H of a group G is normal in G iff the product of any 2 right (left) cosets of H in G is a right (or left) coset of H in G.
51. Factor group.
52. The set G|H of all cosets of a normal subgroup H of a group G is a group under binary operation Ha.Hb = Hab.
53. S.T every factor group of a cyclic group is cyclic.
54. Homomorphism
55. Properties of homomorphism
56. Kernel of homomorphism.
57. Let G be a group & H be a normal subgroup of G; then G|H is homomorphic image of G with kernel H.
58. Fundamental theorem of homomorphism.
59. Some results on isomorphism.
    1. If f:G -> G’ be an isomorphism of group G into G’.
    2. Any infinite cyclic group is isomorphic to the group Z of integers under addition.
60. Rings